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Accuracy of Single- and Multi-Index Models in Stock Investment Portfolios: Study on LQ45 Shares after the Covid-19 Pandemic in Indonesia

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Abstract:
This study discusses the problem of the accuracy of the single- and multi-index models to form a stock investment portfolio. This study aims to reveal information related to the accuracy of the single- and multi-index models to form an optimal stock investment portfolio after the COVID-19 pandemic in Indonesia. The results of the analysis can be used to choose which model is more accurate to form an optimal stock portfolio that can provide returns for investors. This research is devoted to determining the optimal stock portfolio using the single- and multi-index models. The data were collected from stocks included in the LQ45 list for February-July 2022 with 5980 observations consisting of 115 market returns, 1150 industrial sector returns, and 4715 individual stock returns. The analytical method used was regression analysis to predict the expected returns from single- and multi-index models, standard deviation of residuals, and Wilcoxon's differential signed-rank test. The results of the study show that stock investment portfolios based on daily data using the single index model approach and the multi-index model have their respective levels of accuracy with an accuracy rate of 95% (alpha 5%). The main finding is that the multi-index model proved to be more accurate than the single-index model, where the standard deviations of the multi- and single-index models were 0.0215 and 0.0280, respectively.

Keywords: individual stock returns, market returns, multi-index model, single-index model, Wilcoxon's signed-rank test.

股票投资组合中单指数和多指数模型的准确性：印度尼西亚新冠肺炎大流行后 LQ45 股票的研究

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摘要:
本研究讨论了单指标模型和多指标模型构建股票投资组合的准确性问题。本研究旨在揭示与单指数和多指数模型的准确性相关的信息，以在印度尼西亚新冠肺炎大流行后形成最佳股票投资组合。分析的结果可以用来选择哪种模型更准确，以形成能为投资者提供回报的最优股票投资组合。本研究致力于使用单指数和多指数模型确定最佳股票投资组合。这些数据是从 2022 年 2 月至 7 月 LQ45 名单中包含的股票中收集的，有 5980 个观察值，其中包括 115 个市场回报、1150 个工业部门回报和 4715 个个股回报。使用的分析方法是回归分析，以预测单指标和多指标模型的预期收益、残差的标准差以及威尔科克森的微分符号秩检验。研究结果表明，基于日数据的股票投资组合采用单指数模型方法和多指数模型方法具有各自的准确度，准确率达到 95%（alpha 5%）。主要发现是多指标模型比单指标模型更准确，多指标模型和单指标模型的标准差分别为 0.0215 和 0.0280。

关键词：个股回报、市场回报、多指数模型、单指数模型、威尔科克森符号秩检验。

1. Introduction

It is assumed that the COVID-19 pandemic will end in February 2022, and it is hoped that stock performance on the Indonesian capital market will gradually improve. This research was developed from previous research on security investment portfolios, especially stocks. Analysis of stock securities investment portfolios can use market models consisting of the single- and multi-index models (Farrell, 1976; Elton et al., 2014; Roy, 2016). This analysis is important after the Covid-10 pandemic to determine which of the two models is more accurate so that potential investors can plan their investments with maximum returns at certain risks.

Determining an investment portfolio to reduce risk is very important because no single investor likes risk. However, as stated by Hu and Li (2022) that determining an investment portfolio must be an investment fund under normal circumstances, and there is no specific investment object or specific period to determine the portfolio. Determining the investment portfolio is a basic principle in investing in the long term, and this is a form of long-term investment compliance that is carried out repeatedly. In investing, investors hope to get optimal returns at a certain level of risk; therefore, they must form an optimal portfolio to achieve their expectations. Therefore, most investors make various stock investments to meet the expectations of their investment without ignoring the risks that can occur.

An important investment strategy is to balance returns and risks by allocating assets and dividing them into portfolio assets to maximize returns with a certain risk or minimize risk at a certain rate of return. Chen et al. (2022) stated that research on how to manage assets properly was initiated in 1952 by Harry Markowitz. Markowitz's portfolio selection model presented in 1952 has been a theoretical success and is recognized today. However, other models may also work better in different ways when investing in the stock markets every day. Thus, there is a comparison by applying different assumptions and nuances from the results of using different models, such as the single index model. This model was presented by Sharpe (1963) with the aim of simplifying the portfolio model from Markowitz, whose calculations are quite complicated because they involve many variants and covariances. This model assumes that the returns between two or more securities will be correlated, which will move together and have the same reaction to one factor included in the model (Qur'anitasari et al., 2019).

During the COVID-19 pandemic, stock price fluctuations in the capital market were difficult to predict. Historical data are needed to determine real market fluctuations. Comparing various modern portfolio models is necessary to find the right strategy when investing. This action becomes relevant and significant for private investors and portfolio managers so that they can maximize their investment with a certain level of risk. Individual investors and investment managers need to know the empirical facts of how the chosen model limits affect the best results based on historical data after the COVID-19 pandemic. Two models were chosen to make inferences, namely the single-index model (SIM) and multi-index model (MIM). In the SIM, there is an estimated return, and beta as an estimated risk is the only market factor that affects stock returns. The MIM is similar to the Markowitz model with the number of estimated outcomes being n and the estimated variance with the number of covariances being n(n-1)/2 (Chen et al., 2022).

This study uses a market model that is used in multi-index, sectoral indexes. This is based on the argument that market indexes (JCI, IDX30, JII, LQ45, and other indices) are formed from indexes for each sector. Besides that, the data analyzed are daily data based on the argument that stock prices or indexes always change every day, not every month.

2. Literature Review

Some of the current capital market challenges include allocating optimal capital resources, the ability to meet market needs, and investment risk management (Jing et al., 2023). Every investment expects a return that is commensurate with the sacrifices that have been made, so the higher the expected return, the higher the risk of the investment. In this regard, investors are trying to perform an asset placement strategy by composing an investment portfolio to reduce risk through a maximum investment portfolio. A portfolio is a combination of a set of assets, both in the form of real and financial assets owned by investors. The substance of forming a portfolio is to reduce risk through diversification by allocating several funds to other investments that are negatively correlated. Investors can choose returns in the portfolio using the SIM and Markowitz's model (Lestari, 2021).

The main problem in portfolio optimization is selecting optimal assets and securities that are prepared with a certain amount of capital owned (Jing et al., 2023). Although predicting return on investment and minimizing risk appear simple, several methods are used to form an optimal portfolio (Markowitz, 1952; Jing et al., 2023).

The SIM is an approach used to predict market-based individual stock returns. Elton et al. (2014) stated that market-based models are used to analyze investment securities in portfolio formation. Model-based single-index portfolio analysis is an extension of investment portfolio management. The main problem in forming a portfolio is predicting the expected return and standard deviation in one portfolio.

The expected stock returns individually based on the single-index model can be expressed as follows (Elton et al., 2014):

$E(Ri) = \alpha i + \beta iRm + ei$

$E(Ri)$ - expected return on a stock.

αi - the component of security i's return that is independent of the market's performance, a random variable.

Rm - the rate of return on the market index.

βi - a constant that measures the expected change in Ri given a change in Rm.

ei - error term.

The SIM has two assumptions to ensure that the return on an asset is only affected by the return on the market index. The first assumption is that ei is not related to ej for all values of i and j. Thus, $E(ei)$ and $E(ej)$ have values of zero. The second assumption is that ei does not covariate with market index returns. This is due to market index returns (RM) and residual errors (ei), including random variables (Sholehah et al., 2020). Elton et al. (2014) stated that the assumption underlying the single-index model is that changes in stock prices are caused by a change in the market index. Thus, it is necessary to perform tests based on multi-index models.

Sharpe's single-index model has been widely chosen to build an optimal portfolio because of its simplicity and practicality. Several studies have used Sharpe's single index model, which reflects the relationship between risk and return to construct optimal portfolios, including that by Saravanan and Natarajan (2012), who constructed optimal portfolios consisting of four stocks from Nifty 50. In this study, the proportion of investment in each stock is determined based on the cutoff rate of return. Another study was conducted by Nalini (2014) on 15 stocks from various sectors sourced from the S&P index by taking the BSE Sensex as a market index. This research was conducted in India using four selected stocks to construct a portfolio, considering that the capital market in India is still in its development stage. Mishra and Acharya (2015) found that three pharmaceutical company stocks (Glaxo, Sunpharma, and Dr. Reddy) and two banking sector stocks (Axis Bank and Bank of Baroda) are optimal portfolios. Joshi (2015) also conducted the same research to construct an optimal portfolio using data from several stocks from the NSE Nifty. Sangeetha et al. (2021) conducted research on 50 large companies on the Indian National Stock Exchange (Nifty 50), with the results showing that during the study period, there were 42 bullish stocks that gave consistently positive returns and eight stocks that gave negative trends/returns. In accordance with the results of this study, an optimal portfolio is built by choosing 12 stocks above the cutoff rate.

In addition to the single-index model in selecting the optimal portfolio, the multi-index model is also an option for forming an optimal portfolio. As stated by Sorourkhah and Edalatpanah (2022) that many researchers adopt the multi-criteria decision-making (MCDM) approach to conduct optimal portfolios. All researchers try to shape efficiency into the portfolio model, making it challenging to develop an effective portfolio model in a dynamic and uncertain environment (Penadés-Plà et al., 2016).

Multi-index models capture some non-market influences that affect price changes or stock returns. Elton et al. (2014) stated that non-market factors include economic factors and industrial groups or sectors. The industrial sector affects individual stock price changes based on the argument that the market index is also formed from sectoral indexes. Multi-index models introduce extra indexes to capture additional information.

As explained above, the assumption underlying the single-index model is that changes in stock prices are caused by changes in the market index. On the other hand, King (1966) states that changes in stock prices are caused not only by a single index but also the influence of the industrial sector. The industrial sector in the capital market forms a separate index, which is referred to as the sectoral index. These various depositor indices are hereinafter referred to as the multi-index model.

In this multi-index model, it is hypothesized that individual stock returns (Ri) are a function of a set of industry indices. If Ri is the return on stock i, the return on stock i is influenced by the return from the industry index. The equation for multi-index can be formulated as follows (Elton et al., 2014):

$E(R_i) = a_i + b_{i1}I_1 + b_{i2}I_2 + b_{i3}I_3 + \ldots\ldots + b_{ij}I_j + c_i$

$E(R_i)$ - expected individual stock return.

a_i - the component of security i's return that is independent of the industry's index, a random variable.

I_1 - return from Sector 1 index.

I_j - return from sector ke-j index.

b_i - a constant that measures the expected change in

Ri given a change in R_{1-j}.

c_i - error term.

The processes or steps of forming optimal single- and multi-index portfolios are relatively the same. The fundamental difference lies in calculating the expected return where the single index is based on one market index, whereas the multi-index is based on various industry (sector) indices.

The general calculation stages can be performed as follows (Elton et al., 2014):

(1) Calculating individual stock returns (R_i)

$R_{it} = (P_t - P_{t-1})/P_{t-1}$

R_{it} - individual stock return in period *t*.

P_t - current (close) price.

P_{t-1} - previous (close) price.

Return calculations assume that stock returns are obtained from price changes (commonly referred to as capital gains/losses), so dividend yields are not included in the calculation.

(2) Calculating market returns

$R_{mt} = (IHSG_t - IHSG_{t-1})/IHSG_{t-1}$

$IHSG_t$ - composite stock price index period *t* (current).

$IHSG_{t-1}$ - composite stock price index period *t-1* (previous).

In the Indonesian capital market, there are various JCIs, for example, the JCI of all shares listed on the Indonesia Stock Exchange (IDX), ILQ45, which is formed from 45 stocks included in LQ 45, IDX30, and other combined indices. In this study, the combined index used is ILQ45, which assumes that the index of stocks that are actively traded on the IDX is evaluated every semester. ILQ45 reflects the market index for shares actively traded on the IDX. Thus, the market return (R_{mt}) is calculated as follows:

$R_{mt} = (ILQ45_t - ILQ45_{t-1})/ILQ45_{t-1}$

(3) The expected return shares E(Ri) on the single-index model are as follows:

$E(R_i) = a_i + b_iR_{mt} + e_i$

Meanwhile, the expected return of shares $E(R_i)$ in the multi-index model is calculated as follows:

$E(R_i) = a_i + b_{i1}RI_1 + b_{i2}RI_2 + b_{i3}RI_3 + b_{i4}RI_4 + b_{i5}RI_5 + b_{i6}RI_6 + b_{i7}RI_7 + b_{i8}RI_8 + b_{i9}RI_9 + b_{i10}RI_{10} + e_i$

The coefficients b_1 to b_{10} show the beta of each sectoral index, where on the IDX there are 10 industrial sectors from various companies whose shares are traded on the stock exchange. Market returns from Sector 1 to Sector 10 are reflected in equations RI_1 to RI_{10}. Market returns from each sector (RI) are calculated as follows:

$RI_1 = (I_{1t} - I_{1t-1})/(I_{1t-1})$, likewise the market return for the second sectoral index (RI_2) and so on up to RI_{10}.

(4) Risk-free interest rate (RFR)

The risk-free interest rate (RFR) is used as a basis for investor considerations when making investment decisions. In Indonesia, the RFR can be reflected by the interest rate on Bank Indonesia Certificates (SBI) or Republic of Indonesia Government Bond coupons (ORI). Investors tend to invest in SBI or ORI if the expected returns from other investments are the same or even lower than those from SBI or ORI. Thus, SBI or ORI is the minimum interest rate required by investors.

(5) Variance

Variance is the square of the standard deviation, where the standard deviation is the deviation from the mean stock return. The variance error of individual stocks ($Var\ e_i$) is calculated for the variance of stock returns during the observation period. Likewise, the calculation of the market variance ($Var\ e_m$) is also based on the market return from the market index. On a single index, the market variance is calculated based on the return from the LQ45 index; in multi-index, the market variance is calculated based on the return of the sectoral index.

(6) Excess Return to Beta

Excess return to beta (ERB) is the expected return for each stock (ERB_i) obtained from the expected return of stock *i* (ER_i) minus the risk-free interest rate (RFR) divided by beta-stock *i* (b_i) or $ERB_i = (ER_i - RFR)/bi$. This formula applies to the single-index model because b_i is the regression result of one market return. In the multi-index model, the beta coefficient from the regression results has a series of betas from various sectoral indices, so b_i consists of b_{i1} to bin, the number of indices included in the model. The sum from b_{i1} to bin is then used as a divisor of ER_i minus RFR, so the formula for calculating the excess return to beta is as follows:

$ERB_i = (ER_i - RFR)/\sum b_i$

(7) Calculating the value $Ai = \dfrac{((ERi - RFR)*b_i}{Var\ (e_i)}$

(8) Calculating the beta squared (b^2)

(9) B_i is obtained from beta squared divided by variance e_i; $B_i = b^2/Var\ (e_i)$.

(10) Ordering ERB_i from big to small

(11) A_{i-j} is an accumulation of A_i, where A_{i-j} in the first row is obtained from A_i in the first row, A_{i-j} in the second line is obtained from A_{i-j} in the first line plus A_i in the second line, and so on until the last line.

(12) B_{i-j} is an accumulation of B_i, where B_{i-j} in the first row is obtained from B_i in the first row, B_{i-j} in the second row is obtained from B_{i-j} of the first row plus B_i of the second row, and so on until the last line.

(13) C_i is obtained from the intermediate quotient $\{(Var\ e_m)*(A_{i-j})\}/\{(Var\ e_m)*(B_{i-j}+1)\}$, so it is expressed as $C_i = \{(Var_{em}*A_{i-j})\}/\{(Var_{em}*B_{i-j}+1)\}$. Next, the largest C_i (C_i*) is sought as the cut-off rate as the basis for determining the highest risk.

(14) The size of the cut-off point (C*) is the *Ci* value where the last ERB value is still greater than the C_i value.

(15) The stocks that make up the optimal portfolio are those that have an ERB value greater than or equal to the ERB value at point C*.

(16) Calculating the value of X_i obtained from (beta of stock$_{-i}$ divided by variance of stock$_{-i}$) multiplied (ERB minus C*). The calculation can be described as $Xi = (b_i/var_{ei})*(ERB - C^*)$.

Next, W_i is calculated to determine the weight of each stock in the portfolio. The weight or allocation of funds for each share can be proportional, namely each X_i divided by the number of X_i, so that it can be described as $W_i = X_i/\sum X_i$.

3. Methodology

This research uses a quantitative approach, namely closing stock price data, LQ45 index data, and industry sector index data. All the data were taken daily. In addition, it also uses data on the Bank Indonesia interest rate (BI Rate) that applies to the study period, which is 5.25% per year. Non-probability sampling was performed with a purposive sampling technique. The sample criteria are companies that are included in the LQ45 list for February 1–July 31, 2022, and are still on the list in the following period. The number of companies used in this study was 45. However, there were four companies that were not included in the LQ45 list in the following period, namely GGRM, PTPP, TKIM, and WSKT shares, so that the number of companies that were sampled in this study was 41 issuers. From February 1 to July 31, 2022, there were 10 industrial sectors.

This study uses analytical techniques commonly used in determining stock portfolios based on the stages of analysis, from the calculation of stock returns, market returns (single- and multi-index bases), and regression analysis to determine the expected return to the determination and formation of the optimal portfolio. In general, the steps in this study are presented in the research flowchart, as shown in Figure 1.

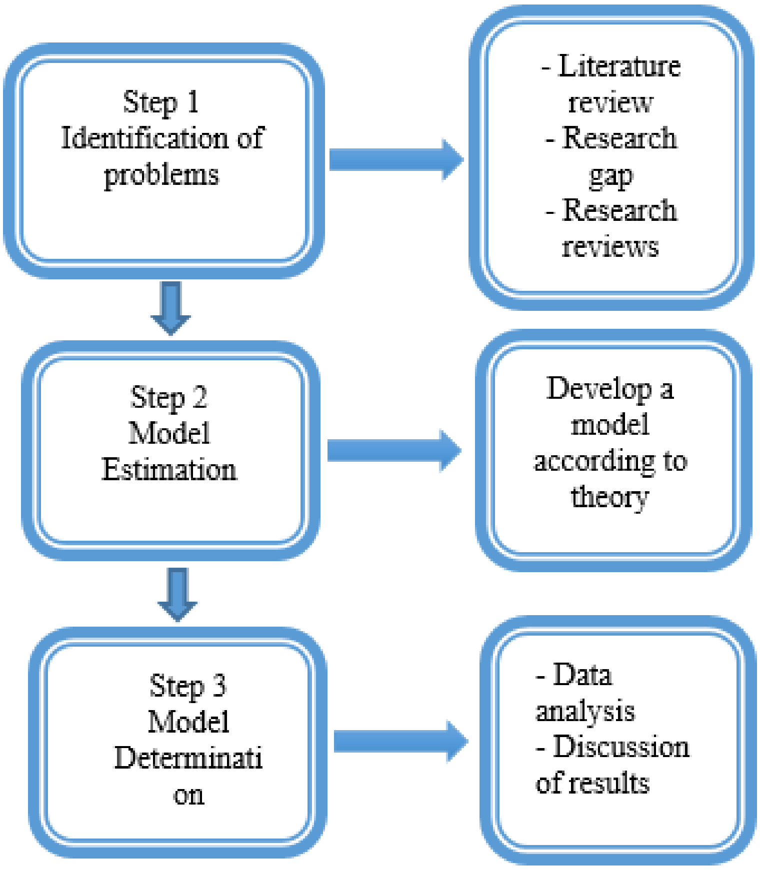

Figure 1. Research flowchart

The analysis technique for the SIM is carried out in the following stages:

(1) The calculation of return for each stock (R_i) is obtained from the following formula:

$R_i = (P_t - P_{t-1})/P_{t-1}$

R_i - stock returns, particularly capital gains/losses.

P_t - stock price (close) today.

P_{t-1} - stock price (close) on the previous day (yesterday).

(2) Calculation of market returns (R_{mt})

$R_{mt} = (ILQ45_t - ILQ_{t-1})/ILQ45_{t-1}$

R_{mt} - market returns.

$ILQ45_t$ - index LQ45 (closed) today.

$ILQ45_{t-1}$ - index LQ45 (close) on the previous day (yesterday).

(3) Calculation of the expected stock return (ER_i)

$ER_i = a_i + b_i R_{mt} + e_i$

ER_i - expected return of individual stocks.

a_i - stock individual alpha.

b_i - regression beta coefficient R_{mt}.

R_{mt} - market returns.

e_i - error term.

(4) Calculation of the excess return to beta: $(ERB) = (ER_i - RFR)/b_i$, where RFR is the risk-free interest rate.

(5) Calculation of A_i and B_i:

$A_i = ((ER_i - RFR)^* b_i/Var_{ei}$, where Var_{ei} is the variance error of individual stocks.

$B_i = b_i^2/Var_{ei}$

(6) Calculation of A_{ij} and B_{ij}, where A_{ij} is the accumulation of A_i, and B_{ij} is the accumulation of B_i.

(7) Calculation of $C_i = (Var_{em}{*}A_{ij})/(Var_{em}{*}B_{ij}+1)$,

where Var_{em} is the variance error of the market, and C_i is the cut-off rate.

(8) The selection of appropriate stocks is based on the size of ERB > C_i.

(9) The determination of the optimal portfolio is based on the proportion of weight W_i of X_i.

$X_i = (b_i/Var_{ei})*(ERB - C_i^*)$, where C_i^* is the largest C_i.

The analysis technique for the MIM outlines the sequence of forming the optimal portfolio relatively the same as that for the SIM. The difference lies in the calculation of the sectoral return index (RIS), R_{mt}, and expected return of individual stocks (ER_i). The sectoral return index (RIS) of 10 indexes (RIS_{1-10}), R_{mt}, and ER_i can be calculated as follows:

$RIS_{1-10} = (IS_{1t} - IS_{1t-1})/IS_{1t-1}) + (IS_{2t} - IS_{2t-1})/IS_{2t-1}) + (IS_{3t} - IS_{3t-1})/IS_{3t-1}) + (IS_{4t} - IS_{4t-1})/IS_{4t-1}) + (IS_{5t} - IS_{5t-1})/IS_{5t-1}) + (IS_{6t} - IS_{6t-1})/IS_{6t-1}) + (IS_{7t} - IS_{7t-1})/IS_{7t-1}) + (IS_{8t} - IS_{8t-1})/IS_{8t-1}) + (IS_{9t} - IS_{9t-1})/IS_{9t-1}) + (IS_{10t} - IS_{10t-1})/IS_{10t-1})$

IS_{1t} - Sector Index 1 in period t (today), IS_{1t-1} - Sector Index 1 in the previous period (yesterday), IS_{2t} - Sector Index 2 in period t (today), IS_{2t-1} - Sector Index 2 in the previous period (yesterday), and so on until the 10th sectoral index.

$R_{mt} = ((RIS_1 + RIS_2 + RIS_3 + RIS_4 + RIS_5 + RIS_6 + RIS_7 + RIS_8 + RIS_9 + RIS_{10})/10)$

$E(R_i) = a_i + b_{i1}RIS_1 + b_{i2}RIS_2 + b_{i3}RIS_3 + b_{i4}RIS_4 + b_{i5}RIS_5 + b_{i6}RIS_6 + b_{i7}RIS_7 + b_{i8}RIS_8 + b_{i9}RIS_9 + b_{i10}RIS_{10} + e_i$

Furthermore, after obtaining optimal portfolio results based on single-index and multi-index models, each model is compared based on the standard deviation of the residuals from the expected return of the selected stocks. If the model produces the smallest residual standard deviation, it is an accurate model. This is based on the argument that the expected return of the two models is based on the regression model, and the best prediction error is the model that produces the smallest residual standard deviation (Gujarati & Porter, 2009; Elton et al., 2014).

3.1. Model Accuracy

The accuracy of the model on the SIM is based on the standard deviation of the residuals from the equation commonly used to predict the expected return of individual stocks (*ERi*) as follows (Elton et al., 2014):

$ER_i = a_i + b_iR_{mt} + e_i$

ER_i - expected individual stock returns.

a_i - stock individual alpha.

b_i - regression beta coefficient R_{mt}.

R_{mt} - market returns.

e_i - error term.

At the level of prediction error on each selected stock in the formation of the portfolio, the residual standard deviation is calculated. If out of 45 stocks, only 10 stocks are selected, the error of these 10 stocks will be calculated for the residual standard deviation. If the residual standard deviation is less than 5% (α = 0.05), the SIM model is declared accurate.

The accuracy of the multi-index model (MIM) is also based on the residual standard deviation of the equation commonly used to predict the expected return of individual stocks (ER_i) as follows (King, 1966; Elton et al., 2014):

$E(R_i) = a_i + b_{i1}RIS_1 + b_{i2}RIS_2 + b_{i3}RIS_3 + b_{i4}RIS_4 + b_{i5}RIS_5 + b_{i6}RIS_6 + b_{i7}RIS_7 + b_{i8}RIS_8 + b_{i9}RIS_9 + b_{i10}RIS_{10} + e_i$

As in the SIM model, the MIM accuracy of the selected stocks in the investment portfolio is then calculated for the residual standard deviation based on the prediction error rate with an alpha of 5% (α = 0.05). If the residual standard deviation is less than 5% (α = 0.05), the MIM model is declared accurate.

Furthermore, to test whether the SIM and MIM are statistically different, the Wilcoxon signed-rank test was used. If it is statistically significantly different at 5% alpha (α = 0.05), then SIM and MIM are declared different; Likewise, if it is not significant at the 5% level, then SIM and MIM are declared not different (same) in the optimal portfolio formation model.

4. Results

Observations of 45 stocks included in LQ45 were performed from February to July 2022 daily. The number of effective observation days (stock exchange working days) during this period was 116 (from February 2 to July 29, 2022). Based on the calculation of returns (stock and market returns) for the periods t and t - 1, the number of the observed returns was 115 (returns for each stock and market returns). Thus, the number of the observations is presented in Table 1.

Table 1. Number of the observations

No.	Objects	Amount	Days	Returns
1	Company	41	115	4.715
2	Index LQ45	1	115	115
3	Sectoral Index	10	115	1.150

Based on Table 1, the number of the observations covered 4,715 individual stock returns, 115 market returns (LQ45), and 1,150 industrial sector returns; therefore, the number of the observations in this study was 5,980.

The results of individual stock beta (b_i), individual stock variance (Var_{ei}), individual stock expected return (ER_i), excess return to beta (ERB), A_i, and B_i show that the stock with the largest ERB is Indo Tambangraya Megah (ITMG) with an ERB value of 0.0134, while the stock with the smallest ERB is Mitra Keluarga Karyasehat (MIKA) with an ERB value of -0.0037 (negative).

In accordance with the determination of proper shares are stocks whose ERB value is greater than the total risk reflected in the cut-off rate (C_i), the result is that stocks number 1 to 10 (ITMG to UNVR) are stocks whose ERB value is greater than C_i. These stocks are included in the optimal portfolio candidate. The list of candidate stocks forming the optimal portfolio is shown in Table 2.

Table 2. List of the candidate stocks and optimal portfolio formation

No.	Stock	*ERi*	Beta	*Var.(ei)*	ERB	*Ci*	*Xi*	*Wi*	SD Res
1	ITMG	0,00588	0,427	0,00099	0,01342	0,00030	5,0260	0,261	0,031
2	AMRT	0,00388	0,752	0,00092	0,00497	0,00063	2,6103	0,135	0,029
3	PTBA	0,00407	0,914	0,00074	0,00430	0,00104	3,1159	0,162	0,025
4	INCO	0,00288	0,818	0,00136	0,00334	0,00115	0,9515	0,049	0,036
5	UNTR	0,00331	0,952	0,00065	0,00332	0,00140	2,2818	0,118	0,023
6	MEDC	0,00177	0,497	0,00111	0,00326	0,00144	0,6740	0,035	0,033
7	MNCN	0,00211	0,647	0,00047	0,00304	0,00155	1,7577	0,091	0,020
8	ADRO	0,00393	1,300	0,00118	0,00291	0,00168	1,2729	0,066	0,031
9	PGAS	0,00214	0,685	0,00060	0,00291	0,00175	1,3012	0,068	0,023
10	UNVR	0,00129	0,537	0,00074	0,00214	0,00176	0,2768	0,014	0,027
						Total/Avg	19,2681	1,0000	0,028

The list of candidate stocks shows that there are 10 stocks suitable for investment because the ERB value is greater than C_i. The weight or proportion in the portfolio as shown in the "W_i" column means that the largest proportion of funds is placed in ITMG shares of 0.261 or 26.1%, followed by PTBA with a proportion of funds of 0.162 (16.2%), AMRT with 0.135 (13.5%), UNTR with 0.118 (11.8%), MNCN with 0.091 (9.1%), PGAS with 0.068 (6.8%), ADRO with 0.066 (6.6%), INCO with 0.049 (4.9%), MEDC with 0.035 (3.5%), and UNVR with 0.014 (1.4%). The average residual standard deviation (SD Res) of the 10 stocks that make up the optimal portfolio is 0.028, where the smallest SD Res is MNCN shares of 0.020, and the largest SD Res is INCO shares of 0.036.

As in the single-index model, the calculation steps for the MIM are as follows. The results of calculating individual stock beta (b_i), individual stock variance (Var_{ei}), individual stock expected return (ER_i), excess return to beta (ERB), A_i, and Bi show that the stock with the largest ERB is Indo Tambangraya Megah stock (ITMG) with an ERB value of 0.0087; the stock with the smallest ERB is XL Axiata (EXCL) with an ERB value of -0.0014 (negative).

In accordance with the determination that a feasible stock is that with ERB value greater than the total risk reflected in the cut-off rate (C_i), the next step is to calculate the ERB, A_{i-j}, B_{i-j}, and C_i. The results of the analysis show that Stocks 1-9 (from ITMG to MNCN) have ERB value greater than C_i. The list of candidate stocks forming the optimal portfolio is shown in Table 3.

Table 3. List of the candidate stocks to form the optimal portfolio

No.	Stock	*E(Ri)*	Beta	*Var (ei)*	ERB	*Ci*	*Xi*	*Wi*	SD Res
1	ITMG	0,0059	,660	0,001	0,00869	0,00045	4,739	0,373	0,022
2	INCO	0,0029	,625	0,001	0,00437	0,00058	1,288	0,101	0,027
3	PTBA	0,0041	1,317	0,001	0,00298	0,00109	2,497	0,197	0,017
4	PGAS	0,0021	,843	0,001	0,00236	0,00121	1,106	0,087	0,019
5	AMRT	0,0039	1,591	0,001	0,00235	0,00142	1,341	0,106	0,025
6	ANTM	0,0014	,597	0,001	0,00211	0,00143	0,265	0,021	0,023
7	UNTR	0,0033	1,524	0,001	0,00208	0,00155	1,184	0,093	0,014
8	MEDC	0,0018	,974	0,001	0,00166	0,00156	0,084	0,007	0,028
9	MNCN	0,0021	1,196	0,000	0,00164	0,00157	0,193	0,015	0,018
							12,699	1,000	0,022

The list of candidate stocks shows that there are 9 stocks suitable for investment because the ERB value is greater than C_i. The weight or proportion in the portfolio as shown in the "*Wi*" column means that the largest proportion of funds is placed in ITMG shares of 0.373 or 37.3%, followed by PTBA with a proportion of funds of 0.197 (19.7%), AMRT with 0.106 (10.6%), INCO with 0.101 (10.1%), UNTR with 0.093 (9.3%), PGAS with 0.087 (8.7%), ANTM with 0.021 (2.1%), MNCN with 0.015 (1.5%), and MEDC with 0.007 (0.7%). The average residual standard deviation (SD Res) of the nine stocks that make up the optimal portfolio is 0.022, where the smallest SD Res is UNTR shares of 0.014, and the largest is MEDC shares of 0.028.

The SIM shows that the smallest residual standard deviation or error (SD Res) is the MNCN share of 0.020, UNTR and PGAS share of 0.023, PTBA share of 0.025, UNVR share of 0.027, AMRT share of 0.029, ITMG and ADRO shares of 0.031, MEDC share of 0.033, and INCO share of 0.036. Meanwhile, the average residual SD is 0.028, meaning that SIM is used accurately to predict the formation of a stock portfolio because it is less than an alpha of 5% ($\alpha = 0.05$).

The multi-index model (MIM) shows that the smallest residual standard deviation or error (SD Res) is the UNTR stock of 0.0136, PTBA stock of 0.0174, and MNCN and PGAS stocks of 0.0180 and 0.0190, respectively. Furthermore, ITMG shares amounted to 0.0216, ANTM (0.0234), AMRT (0.0248), INCO (0.0273), and MEDC of 0.0282. Meanwhile, the average residual SD is 0.0215, meaning that MIM is accurately used to predict the formation of a stock portfolio because it is less than an alpha of 5% (α = 0.05).

In summary, the results of the residual standard deviation test from SIM and MIM are shown in Table 4.

Table 4. Summary of the residual SD results

No.	Stock	SIM	MIM
1	ITMG	0,031	0,022
2	AMRT	0,029	0,025
3	PTBA	0,025	0,017
4	INCO	0,036	0,027
5	UNTR	0,023	0,014
6	MEDC	0,033	0,028
7	MNCN	0,020	0,018
8	PGAS	0,023	0,019
9	ANTM	-	0,023
10	ADRO	0,031	-
11	UNVR	0,027	-

Table 4 shows that eight stocks always included in the optimal candidate portfolio on the two models, SIM and MIM, with a residual SD of less than 0.05. The eight stocks are ITMG, AMRT, PTBA, INCO, UNTR, MEDC, MNCN, and PGAS. Meanwhile, ANTM shares are included in the candidate portfolio on the MIM, and ADRO and UNVR shares are included in the candidate portfolio on the SIM.

The results of the different tests using the Wilcoxon signed-rank test are shown in Table 5.

Table 5. Different test results

Ranks				
		N	Mean Rank	Sum of the Ranks
SIM - MIM	Negative Ranks	1[a]	2.00	2.00
	Positive Ranks	8[b]	5.38	43.00
	Ties	0[c]		
	Total	9		

[a] SIM < MIM
[b] SIM > MIM
[c] SIM = MIM

Test Statistics[a]	
	SIM-MIM
Z	-2.429[b]
Asymp. Sig. (2-tailed)	.015

[a] The Wilcoxon signed-rank test
[b] Based on negative ranks

The results of the Wilcoxon signed ranks test show that it is significantly different. It is shown that the asymp.sig value is 0.015, which is less than α 0.05. Thus, it can be stated that the formation of the LQ45 share portfolio based on SIM and MIM is significantly different.

The formation of an optimal portfolio of stocks included in LQ45 for February–July 2022 shows different results between SIM and MIM. The difference in results lies in the proportion of funds invested and the standard deviation of the residual stocks selected in the investment portfolio. The multi-index model is proven to be more accurate than the single-index model. This is reflected by the standard deviation of the residuals on average at MIM, which is smaller than that at SIM (0.0215 < 0.0280). SIM and MIM accuracy are statistically different at the 5% level. This is indicated by the significant Wilcoxon test results of 0.015 (< 0.05). The results of this study are not in accordance with those by Putra and Dana (2020), who found that the optimal portfolio model using a single-index model has better performance than that using the multi-index model (Markowitz), although there is no statistically significant difference on average return by using a single- and multi-index model (Markowitz).

An important empirical finding of this study is that the results of stock portfolio analysis based on daily data proved to be more accurate than those based on monthly data. This is indicated by the formation of eight stocks that have consistently been selected to form the optimal portfolio using the SIM and MIM approaches. The eight stocks are ITMG, AMRT, PTBA, INCO, UNTR, MEDC, MNCN, and PGAS.

In the SIM model, the largest proportion of funds is ITMG shares (26.1%), and the smallest is UNVR shares of 0.014 (1.4%). Meanwhile, the minimum residual standard deviation is MNCN shares of 0.020, and the largest residual SD is INCO stock (0.036). In the MIM model, the largest proportion of funds is ITMG shares of 0.373 (37.3%), and the smallest is MEDC shares of 0.007 (0.7%). Meanwhile, the smallest residual SD in the MIM model is the UNTR stock of 0.014, and the largest residual SD is the MEDC share of 0.028. These results indicate that the residual SD range for the MIM model is indeed smaller than that for the SIM model. The range of residual SD on MIM is 0.014 (0.028–0.014), whereas the range of residual SD on SIM is 0.016 (0.036–0.020). The smaller the SD residual, the more precise and accurate it is to predict expectations in the coming period, because the smaller the residual (error), the smaller the prediction error (Gujarati & Porter, 2009).

The results of this study agree with and support the findings of research by Qur'anitasari et al. (2019), Fernandez and Kumar (2020), Putra and Dana (2020), Hu and Li (2022) who found that the single-index model had better performance than the multi-index model, even though the difference was not statistically significant. However, the findings from this study are not in accordance with research results by Roy (2016), Chen et al. (2022), and Jing et al. (2023) that multi-index models are better than single-index models.

5. Conclusion

Previous studies have found empirical evidence that the performance of the optimal portfolio model using the single-index model is better than that of the Markowitz model. However, statistically, the average return generated is not significantly different between the two models. This research is based on daily and monthly data, using the single- and multi-index model approaches. The results of the investment portfolio analysis based on the daily data using the single- and multi-index model approaches show that analysis based on daily data is more accurate than that based on monthly data. The single-index model and multi-index model approaches show accuracy at a significance level of less than 5%, however, the single-index model is more accurate than the multi-index model. Another finding is that analyses based on daily data also show more consistency than those based on monthly data.

This research makes an important contribution in the academic field, providing empirical evidence of the application of the concept of portfolio theory in determining stock performance. The multi- and single-index model approaches prove to have high accuracy at determining stock performance. The results of this study did not find a significant difference between the multi- and single-index model approaches. The lesson learned for academics, students, and investors is that capital market conditions determine which approach is more effective. Under high-risk market conditions, it is more appropriate to use a multi-index model approach, whereas under low-risk market conditions, it is more appropriate to use a single-index model approach.

6. Implications, Limitations, and Future Studies

An important contribution of this research is insight into the concept of forming an investment portfolio using the single- and multi-index model approaches. Using these two approaches, we selected stocks that can be formed into an optimal portfolio. There are eight shares: ITMG, AMRT, PTBA, INCO, UNTR, MEDC, MNCN, and PGAS. Of them, there are two shares in the single-index model that are included in the optimal portfolio candidates, namely ADRO and UNVR, and there is one share in the multi-index model, namely ANTM. The results of this study can also be used by investors to determine stock investment portfolios by considering the proportion of funds and residual standard deviation.

Although this research makes an important contribution to the formation of an optimal portfolio, it also has limitations in daily data for one semester after the COVID-19 pandemic and is limited to closing prices, the LQ45 index, and 10 sectoral indices. Thus, the results of this study cannot be extrapolated to include other industries and even manufacturing industries outside Indonesia. In subsequent studies, it is recommended to increase the daily data observation period for two or more semesters. In the multi-index model, other combined indexes can be developed, such as IDX30 and IndexBI40.

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Authors' Contributions

Sunarto oversaw the overall research direction and planning, data collection, analysis, and interpretation.

Ali Ma'sum made substantial contributions to the research design, methodological design of the work, data analysis, overall research supervision, and summarising relevant publications.

Kasmari performed the data analysis, collection, and interpretation.

Bambang Sudiyatno reviewed the literature, synthesized, and collected the data.

References

[1] CHEN, Z., LI, H., LI, Z., & YIN, L. (2022). Analysis of Ten Stock Portfolios Using Markowitz and Single Index Models. *Proceedings of the 7th International Conference on Social Sciences and Economic Development*, 652, 1219–1225.

[2] ELTON, E.J., GRUBER, M.J., BROWN, S.J., & GOETZMANN, W.N. (2014). *Modern portfolio theory and investment analysis*. 9th ed. Hoboken, New Jersey: John Wiley & Sons.

[3] FARRELL, J.L. (1976). *The multi-index model and practical portfolio analysis.* Charlottesville, Virginia: The Financial Analysts Research Foundation.

[4] FERNANDEZ, M., & KUMAR, B.R. (2020). Examination of index model and prediction of beta – A case study examination in IT sector. *International Journal of Economics and Management Systems,* 5, 158-161. http://dx.doi.org/10.5430/afr.v8n2p226

[5] GUJARATI, D.N., & PORTER, D.C. (2009). *Basic econometrics.* 5th ed. New York: McGraw-Hill.

[6] HU, B., & LI, J. (2022). Comparison of the Markowitz and single index model in optimal portfolio. *Proceedings of the 2nd International Conference on Management Science and Industrial Economy Development,* 312-319.

[7] JING, D., IMENI, M., EDALATPANAH, S.A., ALBURAIKAN, A., & KHLIFA, H.A.E.W. (2023). Optimal selection of stock portfolios using multi-criteria decision-making methods. *Mathematics,* 11(2), 415. https://doi.org/10.3390/math11020415

[8] JOSHI, H.S. (2015). Practical application of modern portfolio theory in context to asset allocation by investing in negatively correlated assets reduces your risk. *International Journal of Research in Humanities & Social Sciences,* 3(2), 1-6.

[9] KING, B. (1966). Market and industry factors in stock price behavior. *Journal of Business,* 39(1), 139–190. https://doi.org/10.1086/294847

[10] LESTARI, N.P. (2021). Simulation of optimal portfolio using single index model and Markowitz model on Lq-45 Index Shares for 2018. *Jurnal Bisnis, Manajemen dan Perbankan,* 7(1), 93-140. https://doi.org/10.21070/jbmp.v7i1.880

[11] MARKOWITZ, H. (1952). Portfolio selection. *The Journal of. Finance*, 7(1), 77–91. https://doi.org/10.2307/2975974

[12] MISHRA, R.K., & ACHARYA, S.C. (2015). Optimal portfolio construction in banking and pharmaceuticals sectors in India: An empirical study. *Srusti Management Review,* 8(2), 6-11. Retrieved from https://srustimanagementreview.ac.in/paper_detail.php?id=MjQy

[13] NALINI, R. (2014). Optimal portfolio

construction using Sharpe's single index model – A study of selected stocks from BSE. *International Journal of Advanced Research in Management and Social Science*, 3(12), 72-93. Retrieved from https://indianjournals.com/ijor.aspx?target=ijor:ijarmss&volume=3&issue=12&article=007

[14] PENADÉS-PLÀ, V., GARCÍA-SEGURA, T., MARTÍ, J.V., & YEPES, V. (2016). A review of multi-criteria decision-making methods applied to the sustainable bridge design. *Sustainability*, 8(12), 1295. https://doi.org/10.3390/su8121295

[15] PUTRA, I.K.A.S., & DANA, I.M. (2020). Study of optimal portfolio performance comparison: Single index model and Markowitz model on LQ45 stocks in Indonesia Stock Exchange. *American Journal of Humanities and Social Sciences Research*, 4(12), 237-244. Retrieved from https://www.ajhssr.com/wp-content/uploads/2020/12/ZE20412237244.pdf

[16] QUR'ANITASARI, Q., NUZULA, N.F., & DARMAWAN, A. (2019). Critical analysis of Sharpe, Treynor and Jensen methods in analyzing stock portfolio performance LQ-45 stock studies. *Asia-Pacific Management and Business Application*, 8(2), 89-104. https://doi.org/10.21776/ub.apmba.2019.008.02.2

[17] ROY, S. (2016). Multi-index conditional investment performance measure: An empirical analysis. *International Journal of Financial Management*, 6(3), 14-31. Retrieved from https://www-i-scholar-in.dplt.remotlog.com/index.php/ijfm/article/view/113225

[18] SANGEETHA, S., MADANE, K., & MURALIDARAN, J. (2021). Building an efficient portfolio using Sharpe's single index model (An empirical study with reference to Nifty 50). *Asia-Pacific Journal Management and Technology*, 2(2), 11-21. https://doi.org/10.46977/apjmt.2021v02i02.002

[19] SARAVANAN, A., & NATARAJAN, P. (2012). Optimal portfolio construction with Nifty stocks (An analytical prescription for investors). *Advances in Management*, 5(8), 47-53. Retrieved from https://ssrn.com/abstract=3144198

[20] SHARPE, W.F. (1963). A simplified model for portfolio analysis. *Management Science*, 9(2), 277-293. https://doi.org/10.1287/MNSC.9.2.277

[21] SHOLEHAH, N.A., PERMADHY, Y.T., & YETTY, F. (2020). The Comparison of Optimal Portfolio Formation Analysis with Single Index Model and Capital Asset Pricing Model in Making Investment Decision. *European Journal of Business and Management Research*, 5(4), 1–9. http://dx.doi.org/10.24018/ejbmr.2020.5.4.470

[22] SOROURKHAH, A., & EDALATPANAH, S.A. (2022). Using a combination of matrix approach to robustness analysis (MARA) and fuzzy DEMATEL-based ANP (FDANP) to choose the best decision. *International Journal of Mathematical, Engineering and Management Sciences*, 7(1), 68-80. http://dx.doi.org/10.33889/IJMEMS.2022.7.1.005

参考文:

[1] 陈志、李华、李志、尹丽 (2022)。使用马科维茨和单一指数模型分析十种股票投资组合。第七届社会科学与经济发展国际会议论文集, 652, 1219–1225。

[2] 埃尔顿，E.J.，格鲁伯，M.J.，布朗，S.J.，＆戈茨曼，W.N.（2014）。现代投资组合理论和投资分析。第 9 版。新泽西州霍博肯：约翰·威利父子公司。

[3] 法雷尔，J.L. (1976)。多指标模型和实用投资组合分析。弗吉尼亚州夏洛茨维尔：金融分析师研究基金会。

[4] 费尔南德斯，M.，和库马尔，B.R.（2020）。指数模型检验和贝塔预测——它行业的案例研究检验。国际经济与管理系统杂志，5, 158-161。http://dx.doi.org/10.5430/afr.v8n2p226

[5] 古吉拉特邦，D.N.和波特，D.C. (2009)。基础计量经济学。第五版。纽约：麦格劳-希尔。

[6] 胡本和李杰 (2022)。最优投资组合中马科维茨模型和单一指数模型的比较。第二届管理科学与工业经济发展国际会议论文集，312-319。

[7] JING, D.、IMENI, M.、EDALATPANAH, S.A.、ALBURAIKAN, A. 和 KHLIFA, H.A.E.W.（2023）。使用多标准决策方法优化股票投资组合选择。数学，11(2), 415。https://doi.org/10.3390/math11020415

[8] 乔希，H.S.（2015）。通过投资负相关资产，现代投资组合理论在资产配置中的实际应用可以降低您的风险。国际人文与社会科学研究杂志，3(2), 1-6。

[9] 金，B.（1966）。股票价格行为中的市场和行业因素。商业杂志，39(1), 139–190。https://doi.org/10.1086/294847

[10] 莱斯塔里，N.P.（2021）。使用单一指数模型和马科维茨模型对 2018 年 Lq-45 指数股票进行最优投资组合模拟。商业、管理与银行学杂志，7(1), 93-140。https://doi.org/10.21070/jbmp.v7i1.880

[11] 马科维茨，H.（1952）。投资组合选择。杂志。金融，7(1), 77–91。https://doi.org/10.2307/2975974

[12] MISHRA, R.K. 和 ACHARYA, S.C. (2015)。印度银行和制药行业的最佳投资组合构建：实证研究。斯鲁斯蒂管理评论，8(2), 6-11。检索自 https://srustimanagementreview.ac.in/paper_detail.php?id=MjQy

[13] 纳里尼，R. (2014)。使用夏普单一指数模型构

建最优投资组合——对疯牛病选定股票的研究。国际管理与社会科学高级研究杂志，3(12), 72-93。检索自 https://indianjournals.com/ijor.aspx?target=ijor:ijarmss&volume=3&issue=12&article=007

[14] PENADÉS-PLÀ, V.、GARCÍA-SEGURA, T.、MARTÍ, J.V. 和 YEPES, V. (2016)。应用于可持续桥梁设计的多标准决策方法的回顾。可持续性，8(12), 1295。https://doi.org/10.3390/su8121295

[15] PUTRA, I.K.A.S., & DANA, I.M. (2020)。最优投资组合绩效比较研究：印度尼西亚证券交易所 LQ45 股票的单一指数模型和马科维茨模型。美国人文与社会科学研究杂志，4(12), 237-244。摘自 https://www.ajhssr.com/wp-content/uploads/2020/12/ZE20412237244.pdf

[16] QUR'ANITASARI, Q., NUZULA, N.F., & DARMAWAN, A. (2019)。对夏普、特雷诺和詹森方法在分析股票投资组合表现中的批判性分析 LQ-45 股票研究。亚太管理与商业应用，8(2), 89-104。https://doi.org/10.21776/ub.apmba.2019.008.02.2

[17] 罗伊，S.（2016）。多指数条件投资绩效衡量：实证分析。国际财务管理杂志，6(3), 14-31。检索自 https://www-i-scholar-in.dplt.remotlog.com/index.php/ijfm/article/view/113225

[18] SANGEETHA, S.、MADANE, K. 和 MURALIDARAN, J. (2021)。利用夏普单一指数模型构建高效投资组合（参考漂亮 50 的实证研究）。亚太期刊管理与技术，2(2), 11-21。https://doi.org/10.46977/apjmt.2021v02i02.002

[19] SARAVANAN, A. 和 NATARAJAN, P. (2012)。俏皮股票的最佳投资组合构建（投资者的分析处方）。管理进展，5(8), 47-53。检索自 https://ssrn.com/abstract=3144198

[20] 夏普，W.F.（1963）。投资组合分析的简化模型。管理科学，9(2), 277-293。https://doi.org/10.1287/MNSC.9.2.277

[21] SHOLEHAH, N.A.、PERMADHY, Y.T. 和 YETTY, F. (2020)。最优投资组合形成分析与单指数模型和资本资产定价模型在投资决策中的比较。欧洲商业与管理研究杂志，5(4), 1-9。http://dx.doi.org/10.24018/ejbmr.2020.5.4.470

[22] SOROURKHAH, A. 和 EDALATPANAH, S.A. (2022)。结合使用矩阵方法稳健性分析(玛拉)和基于模糊德马泰尔的心钠素(食品药品监督管理局)来选择最佳决策。国际数学、工程和管理科学杂志，7(1), 68-80。http://dx.doi.org/10.33889/IJMEMS.2022.7.1.005